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Seismic shear response of slab with distributed mass (linear-elastic bay model to story shear)

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ABSTRACT: Common steel structures have concrete floor slabs including cast on stay-in-place steel form deck. In routine seismic design of the slab for in-plane shear, the distributed mass on the slab would be usually treated as the physically lumped structural mass on the bays instead of the slab. Therefore the slab shear response entirely depends on a difference of the opposite bays' displacements and the slab stiffness. Meanwhile as for the structural system including mass distributed on the slab, the seismic behavior has hardly been clarified. This study addresses the maximum local in-plane shear response considering mass distribution on the slab of the system with one-story and its formulation for fundamental seismic design. For the purposes, a variety of linear-elastic structural systems are investigated using time history analysis under earthquake shaking. Then the predictable formulae in case of linear-elastic structural system including bay and slab are newly proposed. Moreover the predictable formulae including previously proposed ones for lumped mass system are verified with the time history analytical results. The comparisons show the validity of the proposed formulae.

1 INTRODUCTION

1.1 *Objectives*

In common, a slab for in-plane shear due to seismicity might be designed against a difference of adjoining bays' displacements and the slab stiffness for in-plane shear. That is to say, a shear deformation of the slab in calculation must be resisted by laterally installed brace members instead. No influence of mass distributed on the slab might be considered. Such calculations suppose that mass distribution on the slab might not cause the additional shear response. The behavior and the formulation of dynamic shear response in-plane problems to the lateral ground shaking for the lumped mass system were previously discussed and reported (Nakamura et al., 2006, 2007). However the investigation of seismic behavior for distributed mass system had hardly been conducted. Therefore objectives of this study are: 1) to obtain fundamental information about local shear response for the distributed mass system of linear-elastic structure, 2) to propose a useful fundamental formula for predicting the maximum local shear response in serviceability limit state design.

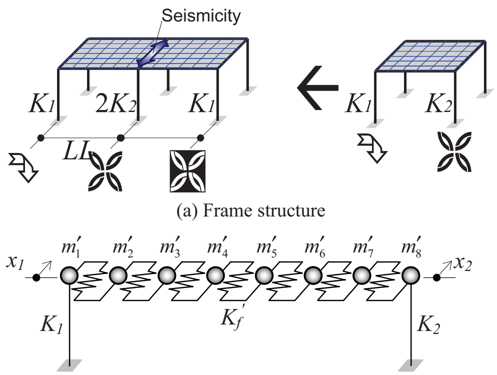
1.2 *Scope*

The scope of this study includes the following:

a. Computation of linear earthquake response of simple structural system including the effect of the mass distribution.
b. Evaluation of results of linear time history.
c. Proposal of a formula for predicting of the maximum local shear response.
d. Evaluation of the proposed formula with the analytical results as well as previously proposed ones for lumped mass system.

2 STRUCTURAL SYSTEMS (ANALYTICAL MODELS)

Consider a frame with 1×2 spans such as previous researches (Nakamura et al., 2006, 2007), according to its symmetry, a simplified model used to analyze in this study consists of two bays with a span (see Fig. 1). The structure has a linear-elastic bay restoring force and story drift relationship as well as the slab. A constant k_1 means a ratio of stiffness of bay 1 K_1 to a sum



(a) Frame structure

$K_f = 7K_f'$

(b) Analytical model

Figure 1. Frame structure and analytical model.

of two bays' stiffnesses K_1 and K_2. The ratio is the following:

$$K_1 = 2K_2\ 4k_1 = \frac{K_1}{K} = \frac{2}{3} \text{ where } K = K_1 + K_2$$

This analytical model assumes the following:

a. Uniform mass distribution on the slab which could be expressed by Table 1 and Fig. 1, and is independent of mass on bay 1.
b. Each mass can move unidirectionally same as the seismicity.
c. The each bay is expressed as mass supported by a shear spring which corresponds to its story drift.
d. Similarly the slab is expressed as 6 mass connected with 7 shear springs which correspond to its in-plane shear deformation. The springs have linear elastic behavior.

The variable structural characteristics in the analytical investigations include the following.

k_f: Slab shear stiffness ratio defined as a ratio of the entire slab stiffness K_f (see Fig. 1(b)) to a sum of stiffness of supporting bays K

$$k_f = \frac{K_f}{K} = 0.1 \sim 1000$$

T_0: Natural period when the slab is rigid

$$T_0 = 0.33,\ 0.67,\ 1.00\ (sec)$$

m_1: Mass ratio of the bay 1 to a sum of mass of the system, to say 1.0

Table 1. Mass distributions.

m_1	0.50	0.67	0.80
M_1	0.50	0.67	0.80
$2M_2$	1.00	0.66	0.40
m'_1	0.125	0.4225	0.65
$m'_1 \sim m'_2$	0.125	0.0825	0.05

$$m_1 = \frac{M_1}{M} = 0.50,\ 0.67,\ 0.80$$

where

$$M_1 = \sum_{i=1}^{4} m_i', M = \sum_{i=1}^{8} m_i', m_i' : \text{distributed mass}$$

The distributed mass $m_1' \sim m_8'$ can be seen in Table 1. For comparison, a lumped mass system might be used as well as the uniformly distributed mass on the slab with a same distance.

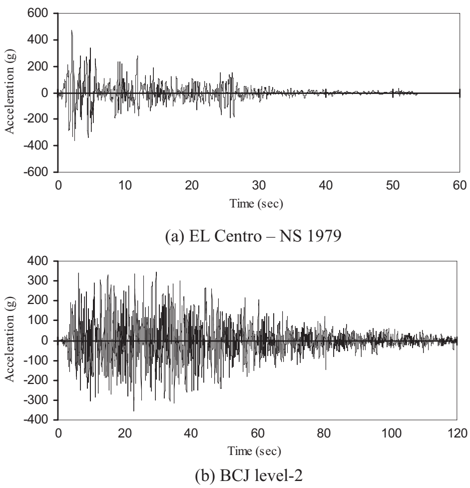
3 TIME HISTORY ANALYSIS

3.1 *Seismic waves*

Applied unidirectional seismic waves are the Imperial Valley earthquake (EL Centro NS 1979) scaled such that its PGV (Peak Ground Velocity) matched 50 kine and the BCJ (Building Center of Japan) level-2 (BCJ 1994), which is an artificial earthquake generated to be compatible with current Japanese seismic code with 50 kine of PGV (see Fig. 2). The integration is performed with a time step of 0.01 seconds during these respective seismic wave durations with two percent of critical damping to the initial stiffness of the bays.

3.2 *Analytical results*

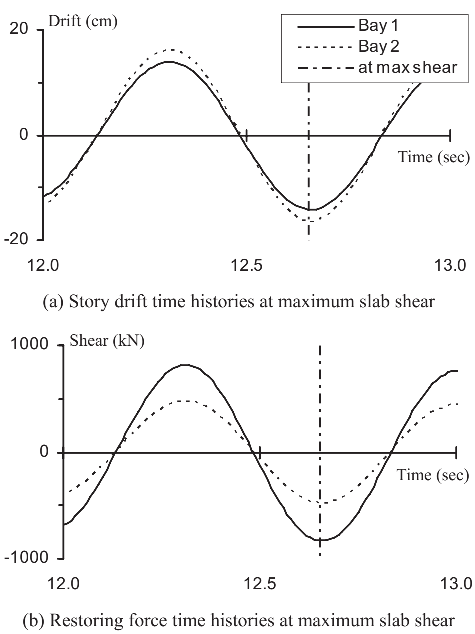
Figs. 3 and 4 indicate earthquake responses of the two bays for EL Centro NS with k_f of 1.0 and 3.0 respectively. These figures include time histories of story drift and restoring force. Whenever the maximum local shear response is generated in linear-elastic structures with slab stiffness ratio k_f equal to or greater than 1.0, the both bays' story drift displacements are always their respective approximate peak values in the cycle. Furthermore, in a range of k_f equal to or greater than 3.0, both bays showed probably same time histories. So the slab can be considered to be rigid. Although the analytical results in case of $T_0 = 0.67$ with EL Centro are illustrated above, the entire results including three different values of T_0, to say 0.33, 0.67 and 1.00 with two different seismic waves,



(a) EL Centro – NS 1979

(b) BCJ level-2

Figure 2. Earthquake excitations.



(a) Story drift time histories at maximum slab shear

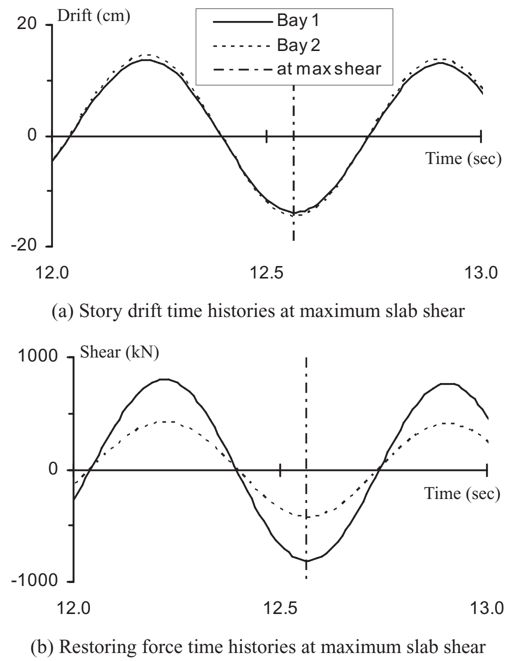
(b) Restoring force time histories at maximum slab shear

Figure 3. Earthquake response of the two bays for EL Centro ($T_0 = 0.67$, m1 = 0.5, $k_f = 1.0$).

have found that the natural period and the wave hardly affect the earthquake response in the slab shear behaviors.

3.3 *Differences of the slab shear response between lumped and distributed mass systems*

Figs. 5(a) and 5(b) display mass displacements and local shear distributions in the slab when

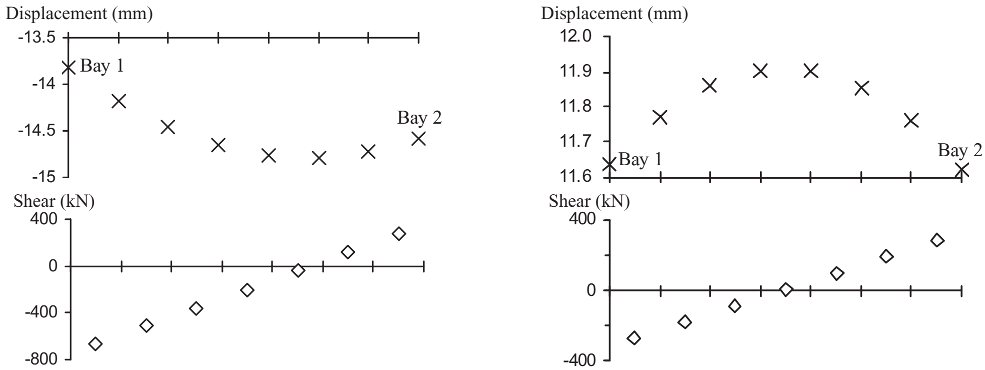


(a) Story drift time histories at maximum slab shear

(b) Restoring force time histories at maximum slab shear

Figure 4. Earthquake response of the two bays for EL Centro ($T_0 = 0.67$, m1 = 0.5, $k_f = 3.0$).

maximum shear response occurred. In these figures, the displacement distributions are not linear. And dynamic shear response variations along the axis normal to the direction of seismic motion can be seen. The shear distributions are not uniform and are approximately proportional to the distance from the bays. That means inertial force applied to the mass on the slab resulted in these behaviors. Although these figures showed only two cases, maximum local shear response could be observed at either shear spring adjacent to the bays due to the inertial force for all the analytical cases including two different seismic waves and three different natural periods with the rigid slab. Fig. 5(b) shows a case that m_1 (= 0.67), the mass ratio of the bay 1 to a sum of mass, equals the constant k_1, a stiffness ratio of the bay 1 to a sum of two bays. In this case displacements of bays 1 and 2 are same, whereas local shear distribution was observed. Then the average shear response was approximately zero. Moreover the local shear response in the middle of the slab is zero. From these facts the inertial force of mass on the slab could be thought transferred to the nearer bay. These findings could not be obtained by the lumped mass system. Needless to say, the maximum dynamic local shear response intensity for the distributed mass system must be equal to



(a) $m_1 = 0.50$ (b) $m_1 = 0.67$

Figure 5. Distributions of mass displacement and local shear response for EL Centro ($T_0 = 0.67$, $k_f = 3.0$).

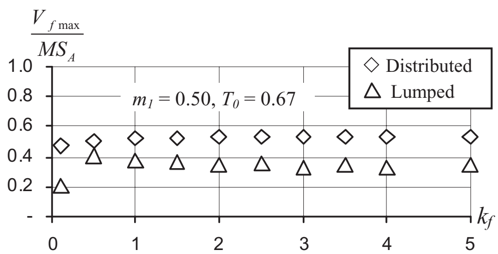


Figure 6. Local shear response comparisons between distributed and lumped mass systems for EL Centro.

or greater than the average value in the slab. Note that the major premise of the lumped mass system is uniform shear response in the slab.

Although in the analyses a sum of the lumped mass equals that of the distributed mass, Nakamura et al. (2006) illustrated that the lumped mass matrix and consistent mass matrix (Archer 1963) for distributed mass in the lumped mass system resulted in probably same maximum slab shear response. Based on the fact, shear response comparisons between the lumped and distributed mass systems are conducted. Fig. 6 shows comparisons of the maximum local shear response between distributed and lumped mass systems with $m_1 = 0.50$ and $T_0 = 0.67$ for EL Centro. In this figure, $V_{f\max}$ and MS_A designate maximum local shear response and a sum of story shear response of the bays, respectively. As for the distributed mass system, inertial force applied to the mass on the slab transferred to the bays could result in additional in-plane shear force in the slab. When the slab stiffness ratio k_f is equal to or greater than 3, increasing the value of k_f could not influence the slab shear response for the distributed mass system as shown in Fig. 6. It is apparent that the lumped mass system could underestimate the maximum local shear response in the slab of the distributed mass. This suggests necessity of a new way to predict that.

4 PREDICTION OF SHEAR RESPONSE

4.1 *Proposal of predictable formulae*

Nakamura et al. (2006, 2007) reported predictable formulae of the shear response based on balance of static force for the lumped mass system without mass on the slab as follows.

Eq. (1) assumes a slab with a certain stiffness not rigid, meanwhile Eq. (2) dose a rigid one. In case of the rigid slab, every single mass displacement must be always equal as well as the absolute acceleration. Thus each inertial force of the mass is proportional to the mass quantity. Moreover a story shear ratio of the bays is same as their elastic stiffness ratio. From Eq. (2), maximum sum of the story shear of the bays could attribute to the maximum shear response of the slab. When slab shear stiffness ratio k_f is sufficient in Eq. (1), this formula could be approximately equal to Eq. (2). If the slab stiffness ratio k_f is infinite, these formulae are identical.

$$V_{f\max} = K_f \left|x_1 - x_2\right| = \frac{k_f \left|m_1 - k_1\right|}{k_1(1-k_1)+k_f} MS_A \tag{1}$$

$$V_{f\max} = \left|m_1 - k_1\right| \left(V_1 + V_2\right)_{\max} \tag{2}$$

where K_f designates entire slab shear stiffness, x_1 and x_2 designate story drifts of bays 1 and 2, V_1 and V_2 designate each bay's story shear response, respectively. Then in the design procedure, $(V_1 + V_2)_{\max}$ is given as MS_A (S_A: design spectral response acceleration).

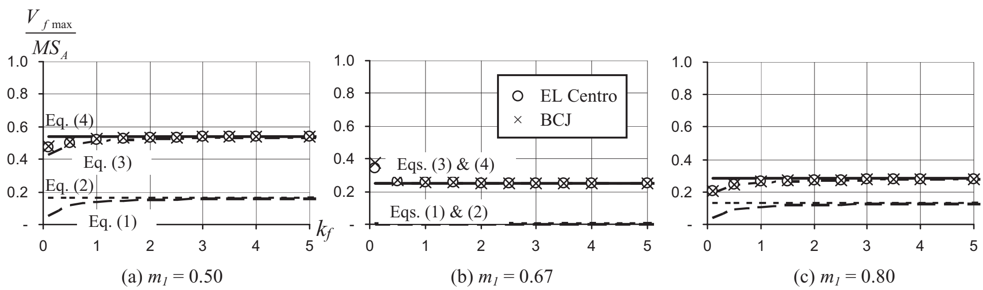


Figure 7. Maximum local shear response comparisons between analytical and predicted values ($T_0 = 0.67$).

When the mass ratio m_1 and the stiffness ratio k_1 of bay 1 are equal, both above formulae must yield zero slab shear response. On the other hand, these formulae could not take into account additional shear response due to inertial force of the distributed mass. Assume that inertial force of the mass might be transferred to nearer bay, the following Eq. (3) is proposed here. Considering practicality and convenience of the seismic design procedure, Eq. (4) is also available to predict the maximum local shear response, based on Eq. (2).

$$V_{f\max} = K_f \left| x_1 - x_2 \right| + \max\left(\sum_{i=2}^{n/2} m_i', \sum_{i=n/2+1}^{n-1} m_i' \right) S_A$$
$$= \frac{k_f \left| m_1 - k_1 \right|}{k_1(1-k_1)+k_f} MS_A + \max\left(\sum_{i=2}^{n/2} m_i', \sum_{i=n/2+1}^{n-1} m_i' \right) S_A \quad (3)$$

$$V_{f\max} = \left| m_1 - k_1 \right| MS_A + \max\left(\sum_{i=2}^{n/2} m_i', \sum_{i=n/2+1}^{n-1} m_i' \right) S_A \quad (4)$$

where n and m_i' designate number of mass and distributed mass respectively.

4.2 *Verification of predictable formulae for slab subjected to shear caused by seismicity*

Figs. 7(a), 7(b) and 7(c) indicate maximum local shear response comparisons between analytical and predicted values with T_0 of 0.67 for the distributed mass system. In these figures ○ and × symbols designate analytical results for EL Centro and BCJ respectively. When the slab stiffness ratio k_f is equal to or greater than 3, increasing the value of k_f could not influence the analytical slab shear response as well as Fig. 6. The previous Eqs. (1) and (2) could underestimate the maximum local shear response as shown in the figure. Especially it is noted that in any cases of $m_1 = 0.67$ both previous formulae generate no shear response in-plane of the slab.

By taking account for inertial force applied to the mass not only on the bays but also on the slab, in addition to difference of the story drift between two bays of both sides of the slab, the maximum local shear response in the slab could be predicted appropriately by Eq. (3). When the slab stiffness ratio k_f to the adjoining bays is equal to or greater than 3, Eq. (4) is also available for the prediction as well as Eq. (3). In routine design, Eq. (4) is preferable to Eq. (3) for its simplicity. That is to say, it is not required to know a difference of the displacement between the bays for calculation in Eq. (4).

In design procedure, it is also required to evaluate shear capacity of slab to calculate the DCR (Demand Capacity Ratio). To evaluate the shear capacity of the slab, some kinds of loading test have been generally conducted. However in these previous static or dynamic loading tests, variation of the shear response in the slab might to be hardly supposed. Different shear distributions might yield different shear mechanisms. Thus the differences of the shear mechanism between actual dynamic response and these loading tests could be possibly thought. But it is hardly justified to evaluate accurately the differences of the shear mechanism. Therefore based on the mass distribution, statically loading test or shaking table test might be desired. Moreover it is important for statically loading tests to evaluate accurately dynamic mechanical characteristics of used slab material.

5 CONCLUSIONS

In this study a variety of time history analyses were conducted in order to investigate the dynamic shear response behavior of the slab with distributed mass, which is supported by bays with elastic-linear restoring force characteristics.

The results and conclusions of the analytical studies presented in this paper may be summarized as follows:

a. Local shear response of the distributed mass system could differ from that of the lumped mass system.
b. The distributed mass system showed dynamic shear response variations along the axis normal to the direction of seismic motion, according to the mass inertial forces on the slab.
c. The maximum local shear response could be observed at either shear spring adjacent to the bays for all the analytical cases.
d. Inertial force of mass on the slab might be transferred to nearer bay since the local shear response is promotional to the distance from the bays.
e. The lumped mass system might underestimate the maximum local shear response for the distributed mass system.
f. The previously proposed Eqs. (1) and (2) could underestimate the maximum local shear response of the distributed mass system.
g. The maximum local shear response in the slab could be predicted appropriately by newly proposed Eq. (3). When the slab stiffness ratio k_f to the adjoining bays is equal to or greater than 3, Eq. (4) is also available for the prediction.

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NOTATION

The following symbols are used in this paper:

K = a sum of stiffness of 2 bays
k_1 = stiffness ratio of bay 1 (K_1/K)
K_1 = stiffness of bay 1
K_2 = stiffness of bay 2
k_f = slab shear stiffness ratio to the supporting bays (K_f/K)
K_f = entire slab stiffness which is a sum of K_f'
K_f' = stiffness of distributed shear springs (see Fig. 1)
M = a sum of all mass
M_1 = a sum of mass deemed to be supported by bay 1
m_i' = distributed mass
S_A = story shear response acceleration
T_0 = natural period with the rigid slab
V_1 = story shear force of bay 1
V_2 = story shear force of bay 2
x_1 = story drift of bay 1
x_2 = story drift of bay 2

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